

Kinematic Analysis Of Double Pendulum For Exoskeleton Walking Aid

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Abstract— The exoskeleton concept, initially developed to assist soldiers in carrying heavy loads, has also been explored for medical applications in patients with mobility impairments. In this study, a two-degree-of-freedom model is proposed, focusing on the hip and knee joints. The Lagrange formulation is employed to determine the angles at these joints. These two joints play a big role in designing the robot as a new link that connects both thigh and shank for the new exoskeleton will be introduced and new equation utilizing trigonometric functions is introduced to quantify the behavior of the new link. The results obtained from the Lagrange formulation and the trigonometric approach are compared to assess the practicality of the new link in real-life scenarios. MATLAB/Simulink simulations are conducted to evaluate the response of both approaches. Additionally, the exoskeleton design is developed using the SOLIDWORKS program, with certain components fabricated through 3D printing. Experimental testing of the exoskeleton is performed, utilizing a potentiometer connected to an Arduino to measure the knee angle. The obtained data is compared with the results obtained from the trigonometric function, revealing similar patterns between the simulated and experimental data. These findings indicate that the exoskeleton design is viable for practical implementation.

Keywords—double pendulum, exoskeleton, kinematic analysis

I. INTRODUCTION

The human walking pattern has been studied for decades, benefitting everyone, including people with disabilities. Many intelligences and researchers utilize this walking mechanism as an inspiration to create an exoskeleton, which is a robot that can assist with lower-limb walking [1, 2]. This idea arose from patients who require rehabilitation due to disabilities such as stroke, which affect the residual function and disrupt their walking pattern [3, 4]. However, many people wonder why an exoskeleton is necessary when there are treatments that can help patients with walking problems. Consistent practice is required for patients to see rapid changes in their walking patterns. However, with therapy, they must wait for their appointment before they can practice walking. As the number of people with impairments rises each year and there are not enough therapies to cure the patients, this process could take a little while, resulting in numerous therapy sessions.

Exoskeletons were initially studied for use in the military, where they are utilized to improve soldiers'

capability to transport loads [5]. The notion of this exoskeleton that soldiers use has been refined to help individuals in need. For example, the Japanese firm CYBERDYNE Inc. created the hybrid assistive limb (HAL), an exoskeleton technology, to help the elderly with lower limb functional degeneration or patients with lower limb paralysis [6]. The exoskeletal system has existed previously, albeit today, more advanced models are available. Instead of waiting years to develop the proper walking pattern like a regular person, many diseases are treatable with significant improvement thanks to wearable robots. Exoskeletons such as the Indego (Parker Hannifin Corp. USA), Ekso (Ekso Bionics, USA), and Atalante (Wandercraft, France) are intended to assist patients in walking more naturally [7]. Previously, they calculated the value of the angles using a formula with a large number of degrees of freedom, such as 10-degree-of-freedom [5]. Operating the exoskeleton also involves extensive use of actuators. The addition of actuators will place too much weight on the robot, making it more difficult for people with lower limb difficulties to move it because the exoskeleton requires torque to function effectively.

Therefore, in this project, only one actuator will be implemented to move the knee and hip, making it less expensive than other exoskeletons. On top of that, the patients only need to provide a small number of torques to the exoskeleton. As the exoskeleton is so light, patients can wear it without the assistance of other people, allowing them to practice their walking pattern whenever they wish without the risk that they will not be able to handle the robot on their own. This exoskeleton also solely considered the angle at the knee and hip. Simply put, this exoskeleton is less complicated than the present one since it only considers two degrees of freedom at the hip and knee. As a result, an attachment that provides the optimal angle for a walking pattern is essential. All measurements will be fixed, and the linkage model will be designed in SOLIDWORKS. This design will be centered on the hip-knee angle since when the hip moves, the knee will automatically follow in response.

II. DERIVATION OF MATHEMATICAL MODEL FOR EXOSKELETON

A. Modeling of exoskeleton using Lagrange

Based on the previous literature reviews conducted, it has been identified that there are two commonly utilized approaches for deriving a mathematical model. Nevertheless, the Lagrange method offers a simpler solution compared to Newton's Law. Additionally, conducting experiments on the knee modelling method has proven effective in implementing the approaches proposed by previous researchers. While most researchers have suggested a high degree of freedom (DOF) for the model, it is not suitable for this project due to unnecessary parameters for simplified model. As a result, only a 2-DOF model is considered. To ensure an accurate representation of the leg, an appropriate method is selected using Lagrange formulation. This exoskeleton is using a double pendulum which is need to consider two links that attach to the length of hip and knee. They also combined with vertical angle of the length and those things will be used in the derivation of the mathematical modelling. For the derivation, Lagrange formulation is used to find the angle for the knee and hip. This formulation gives the equation to find the angle using the suitable parameters of the model. Figure 1 provides a visual representation of the labelled parameters for the leg mechanism. The parameters include m_1 , which represents the mass between the hip and the knee, m_2 , which represents the mass between the knee and the ankle, l_1 , which denotes the length measured from the hips to the knees, and l_2 , which signifies the length measured from the knee to the ankle. Table 1 presents the reference parameters used to determine these parameters.

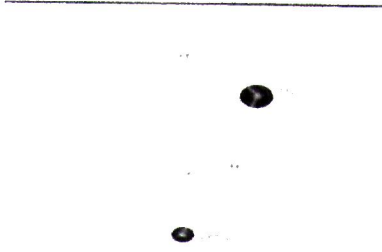


Figure 1: The label parameter of hip and knee.

Table 1: Parameters that is used for the exoskeleton

Parameter	Value
m_1	1.5 kg
m_2	1.25 kg
l_1	0.09 m
l_2	0.14 m

A double pendulum is composed of two pendulums connected to each other. In Figure 1, there is a double-bob pendulum with masses labelled as m_1 and m_2 , attached to link of lengths labelled as l_1 , and l_2 , respectively. The pendulum is characterized by the vertical angles of the two links, denoted as θ_1 and θ_2 . The gravitational acceleration, represented by the symbol g , is a known constant and can be expressed using the following equation. The displacements

in x and y directions for both l_1 and l_2 are written in equations (1) to (4).

$$x_1 = l_1 \sin \theta_1 \quad (1)$$

$$y_1 = -l_1 \cos \theta_1 \quad (2)$$

$$x_2 = l_1 \sin \theta_1 + l_2 \sin \theta_2 \quad (3)$$

$$y_2 = -l_1 \cos \theta_1 - l_2 \cos \theta_2 \quad (4)$$

Equations (1) to (4) represent the positions of l_1 and l_2 in the x and y directions. The equations of motion for the exoskeleton using double pendulum are derived using Lagrangian formulation. Generally, the Lagrangian is a mathematical function that describes the difference between the kinetic and potential energies that can be written as follows:

$$L = T - U \quad (5)$$

Where, T represents the total kinetic energy and U represents the total potential energy of the mechanism. The kinetic energy T of the double pendulum is the sum of the kinetic energy of each individual pendulum. Similarly, the potential energy U is the sum of the potential energy of each individual pendulum. The total kinetic and potential energies can be written as:

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2) \quad (6)$$

$$U = m_1 g y_1 + m_2 g y_2 \quad (7)$$

The final equation of the Lagrange is written (8) as follow:

$$L = T - U = \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 (l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos \theta_1 - \dot{\theta}_2^2) + m_1 g l_1 \cos \theta_1 + m_2 g l_2 \cos \theta_2 \quad (8)$$

By using the equation of motion, the equation of the angular motion that have been derived are then extended to obtain angular motions for both hip and knee as presented in equation (9) and (10).

$$\ddot{\theta}_1 = \frac{-m_2 l_1 l_2 \cos^2 \theta_1 \dot{\theta}_2^2 - m_2 l_1 g \sin \theta_1 - m_2 l_2 \dot{\theta}_2^2 \sin \theta_1 - m_1 g \sin \theta_1}{(m_1 + m_2) l_1} \quad (9)$$

$$\ddot{\theta}_2 = \frac{(5l_1^2 + m_1 l_1^2 \dot{\theta}_1^2 \sin^2 \theta_1 - \dot{\theta}_1^2 - m_1 l_1 g \sin \theta_1 - m_2 l_1^2 \dot{\theta}_1^2 \cos \theta_1 - \dot{\theta}_1^2)}{m_2 l_2} \quad (10)$$

B. Additional Linkage Using Trigonometric

In order to use additional link, a trigonometric function will be considered. This additional linkage introduces additional variables and relationships that need to be considered in the simulation model. To represent the interactions and constraints of the new linkage, trigonometric equations are employed. These equations provide a mathematical framework to symbolize the geometric relationships between the various components of the system. By incorporating trigonometric equations, the simulation accurately captures the complex interplay between the linkages and enables a more comprehensive analysis of the

system's behavior. This approach facilitates the exploration of different configurations and allows for a detailed investigation of the impact of the additional linkage on the overall system dynamics.

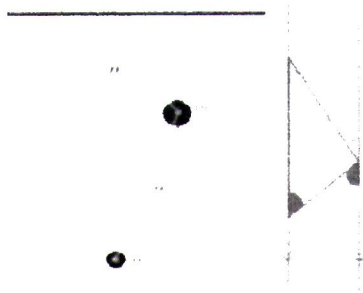


Figure 2: The trigonometric of the hip and knee

In Figure 2, there is a double-bob pendulum with masses labelled as m_1 and m_2 , attached to links of lengths labelled as l_1 , and l_2 , respectively and l_3 as the additional link that connect both l_1 and l_2 . The pendulum is characterized by the vertical angles of the two links, denoted as θ_1 and θ_2 . But for the trigonometric equation, the variable θ_1 is derived from the previous result obtained through the Lagrange equation. This approach ensures the continuity and consistency of the simulation by incorporating the calculated value of θ_1 as an input for the trigonometric equations. The trigonometric equation can be calculated as:

$$E_{2new} = l_1^2 + l_3^2 - 2(l_1)(l_3) \cos\theta_1 \quad (11)$$

The angle of θ_{2new} need to be 66.23 degree (± 5.42) [8], so by using ratio, the best ratio for the angles at hip and knee can be obtained.

$$\theta_{2new} = \frac{\theta_1}{0.7l_1} [l_1^2 + l_3^2 - 2(l_1)(l_3) \cos\theta_1] \quad (12)$$

Once the necessary equations have been derived from the trigonometric representations, they are seamlessly integrated into MATLAB/Simulink for simulation purposes. The obtained equations are implemented within the Simulink environment to model the behavior of the system. This integration allows for the visualization and analysis of the system's dynamics in a simulated environment. The simulation by using the parameters of θ_1 , l_1 , and l_3 where θ_1 is obtained from the previous Lagrange equation. Through the simulation in MATLAB/Simulink, the effects of the trigonometric equations on the overall system behavior can be observed and analyzed. This provides valuable insights into the performance, stability, and other characteristics of the system under investigation.

The utilization of Simulink enables the exploration of the interplay between the previously obtained θ_1 angle and the newly derived equation representing the angle of the knee in the exoskeleton system with an additional link. By integrating the results from the previous calculation into the Simulink model, it becomes possible to determine the angle of the knee based on the system's dynamics and the trigonometric equation. During the simulation, it is essential

to ensure that both the newly calculated angle and the previous result fall within an acceptable and practicable range. This range is determined by the physical limitations and constraints of the exoskeleton system. By imposing these constraints, the Simulink model provides a more realistic representation of the system's behavior and ensures that the obtained results are both feasible and applicable in real-world scenarios. The evaluation of the simulation results within the acceptable range is crucial for assessing the effectiveness and practicality of the exoskeleton system with the additional link. By considering the limitations and constraints during the simulation, the Simulink model serves as a valuable tool for refining the design, optimizing the performance, and ensuring the overall feasibility of the exoskeleton system with the additional link.

C. Design of Exoskeleton

After successfully completing the simulation phase and ensuring the accuracy and feasibility of the exoskeleton system, the next step is to proceed with the design phase using SOLIDWORKS. SOLIDWORKS provides a powerful platform for creating and visualizing detailed 3D models of the exoskeleton. By utilizing SOLIDWORKS, precise measurements, materials, and components can be incorporated into the design to accurately represent the real-world exoskeleton. Once the design has been selected and confirmed, the fabrication stage follows. Fabrication involves transforming the virtual design into a physical product. This step typically involves using specialized manufacturing techniques such as 3D printing, or custom fabrication processes. The fabrication stage aims to create a functional prototype or a final version of the exoskeleton. With the fabricated exoskeleton in hand, it can be tested with actual human subjects to evaluate its performance. This includes assessing factors such as comfort, range of motion, support, and the interaction between the user and the exoskeleton. The testing phase provides valuable insights into the real-world performance of the exoskeleton and allows for further refinements or optimizations based on the observed results.



Figure 3: The exoskeleton design

Figure 3 illustrates the exoskeleton design created using SOLIDWORKS. The design showcases the integration of an additional linkage, extending from the hip to the knee joint. This linkage plays a vital role in facilitating the movement of

the 38 lower limb. It is connected to the exoskeleton using bearings and a *c*-section, which contribute to smooth and controlled motion. The inclusion of the bearing and *c*-section in the design serves to enhance the functionality of the exoskeleton. The bearing ensures proper alignment and support for the linkage, enabling it to move smoothly up and down during the wearer's activities. This feature enhances the range of motion and flexibility of the lower limb, improving the overall performance of the exoskeleton. Moreover, the design of the exoskeleton is specifically tailored to ensure that the knee joint moves in coordination with the hip joint. This design consideration ensures that the exoskeleton mimics natural human movement. By enabling the knee to move in response to the hip's motion, the exoskeleton provides a more seamless and intuitive experience for the user. The design also prioritizes synchronized motion between the hip and knee joints, resulting in an exoskeleton that closely emulates natural human movement patterns.

III. EXPERIMENT SETUP

After the completion of the design phase and simulation using SOLIDWORKS and MATLAB/Simulink, the next crucial step involves fabricating the exoskeleton and conducting experimental tests to validate the simulated results. The fabricated exoskeleton serves as the physical embodiment of the design, allowing for actual evaluation and measurement of the angles. The experimental setup involves attaching sensors, such as potentiometers, to key joints of the exoskeleton, including the hip and knee. These sensors are carefully calibrated to accurately capture the angles of the exoskeleton during various movements and activities. By monitoring the angle readings from the sensors, the experimental setup enables a direct comparison between the measured angles and the angles predicted by the simulation. To ensure reliable and consistent data acquisition, the sensors are connected to appropriate data acquisition systems, namely Arduino board. These systems collect and process the sensor data in real-time, providing immediate feedback on the actual angles of the exoskeleton. By combining the fabricated exoskeleton, calibrated sensors, and robust data acquisition systems, the experimental setup provides a reliable means to compare the actual angle measurements with the simulated angles. This validation process is crucial in assessing the accuracy and effectiveness of the exoskeleton design and simulation, enabling any necessary refinements or adjustments to optimize its performance.

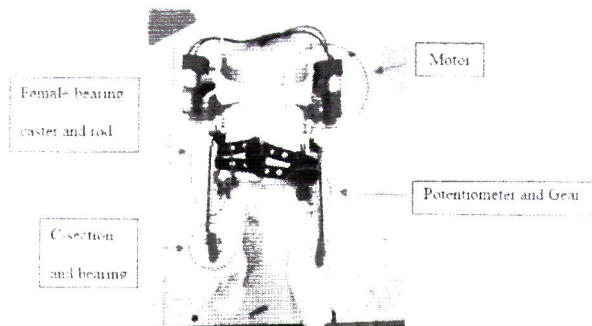


Figure 4: Fabricated exoskeleton.

Figure 4 shows the fabricated exoskeleton that utilized in the experimental phase. This exoskeleton serves as the physical implementation of the design and simulation, and it is specifically tailored for the intended experiment. Notably, the motor depicted in the figure plays a critical role in providing torque to the hip joint, enabling controlled movement to the desired angle. To accurately measure the angles during knee movement, the exoskeleton incorporates a potentiometer and gear system. This combination allows for precise angle readings, providing valuable data for analysis and comparison with the simulated values. Additionally, the inclusion of a *c*-section and bearing mechanism ensures coordinated motion between the hip and knee joints. This mechanism enables the knee to move synchronously as the hip moves, contributing to a more natural and realistic exoskeleton experience.

Furthermore, the exoskeleton design includes a female bearing and rod assembly, which serves as the connection between the hip and knee joints. This arrangement facilitates the transfer of motion and forces between the two joints, ensuring seamless coordination during movement. Collectively, the various components and mechanisms illustrated in Figure 5 demonstrate the functional aspects of the fabricated exoskeleton. The motor, potentiometer, gear system, *c*-section, bearing, female bearing, rod, and caster all contribute to the overall performance and functionality of the exoskeleton, enabling precise movement, accurate angle measurement, and synchronized motion between the hip and knee joints.

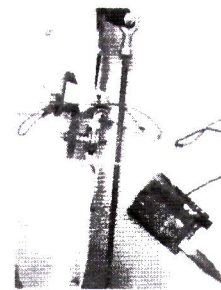


Figure 5: Arduino setup for the exoskeleton.

The setup shown in Figure 5 is designed specifically for acquiring the angle readings at the knee joint. To facilitate this process, an Arduino board is utilized and connected to a laptop, which runs the necessary code for recording the angle readings. The Arduino serves as the interface between the sensors or potentiometers at the knee joint and the data recording software. Upon receiving the angle readings from the sensors, the Arduino records the data in real-time, capturing the precise angles as the exoskeleton undergoes movements and tasks. The recorded angle data is then transferred to a spreadsheet program, namely Microsoft Excel, for further analysis and visualization. By transferring the data to Microsoft Excel, it becomes easier to organize and manipulate the angle readings. This enables the generation of graphs and visual representations of the recorded angles, facilitating a comprehensive analysis of the experimental results. The graphs serve as valuable tools for comparing the actual angle readings with the simulated values obtained from the earlier simulation. The comparison between the experimentally measured angles and the simulated values allows for a thorough assessment of the accuracy and validity of the simulation model.

IV. SIMULATION AND EXPERIMENTAL RESULTS OF THE PROPOSED EXOSKELETON

In this section, a concise discussion of the obtained results will be provided, along with an analysis of the collected data from this project. The simulation conducted using MATLAB Simulink is presented, showcasing the exoskeleton's response during the walking phase. The simulation results will serve as a valuable reference to evaluate the exoskeleton's performance and validate the model estimation. Furthermore, to enhance the credibility and reliability of the findings, the simulation result will be compared with the experimental data from the previous study [9]. This comparison is enabled to assess the accuracy and effectiveness of the model by validating it against established experimental results. Figure 6 presents a comparison between the graphs that is obtained from Simulink using the Lagrange equation and the trigonometric function. The graphs illustrates that the maximum angle achieved in both cases is different, but the time taken by both graphs to complete a single cycle remains similar. Additionally, both results show a similar pattern and they also share a common underlying behavior, even though different mathematical approaches were used.

Figure 6: Knee angle responses between Lagrange equation and trigonometric function.

Next, Figure 7 shows the comparison of knee angle responses from the trigonometric, experimental and published result set as a benchmark. From the result, it can be seen that both trigonometric and experimental responses have similar maximum amplitude and the time taken to complete one cycle process. However, the experimental response exhibits a noticeable deviation at the second and third cycles as compared to the trigonometric function. Next, when comparing the amplitude of the experimental response and the trigonometric function response, both can be seen to have a similar maximum and minimum amplitude despite the presence of the deviation. But overall, the graphs show that, both have the similar trend of cycle and the time taken to complete one cycle also remain similar.



Figure 7: Comparison of angle responses for all cases.

V. CONCLUSION

In this study, the derivation for 2-DOF of the hip and knee models using the Lagrange method was completed in order to understand the response of the leg while walking. The Lagrange method and trigonometric function are compared to observe whether both approaches are having the similar responses. These two approaches are simulated using MATLAB/Simulink. Other than that, the exoskeleton is validate using potentiometer and Arduino which is also to see the approach model using trigonometric function is valid and practicable or not. The design of the exoskeleton is done by SOLIDWORK software which is to get the best design before fabricating the real exoskeleton. From the experimental result, it can be seen that the Trigonometric model managed to track the experimental data as well as the result obtained from the published data with acceptable error.

ACKNOWLEDGMENT

The research has been carried out under Self-Fund Research Grant, SF0141-UJPNM/2023/SF/TK/3 at Faculty of Engineering, National Defence University of Malaysia.

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