SEMARAK ILMU
PUBLISHING

Journal of Advanced Research in Applied Sciences and Engineering Technology

Journal homepage:
https://semarakilmu.com.my/journals/index.php/applied_sciences_eng_tech/index
ISSN: 2462-1943

The Two Phase Generalized Mean Model for Image Segmentation

Nurul Asyiqin Mohd Fauzi[1], Mazlinda Ibrahim[2*], Hoo Yann Seong[2], Abdul Kadir Jumaat[3], Lavdie Rada[4], Haider Ali[5]

[1] Pusat PERMATA@Pintar Negara, Universiti Kebangsaan Malaysia, 43600 Bangi, Malaysia
[2] Centre for Defence Foundation Studies, National Defence University of Malaysia, 57000 Kuala Lumpur, Malaysia
[3] School of Mathematical Sciences, College of Computing, Informatics and Media, Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia
[3] Institute for Big Data Analytics and Artificial Intelligence (IBDAAI), Universiti Teknologi MARA (UiTM), 40450 Shah Alam, Selangor, Malaysia
[4] Biomedical Engineering Department, Bahcesehir University, Besiktas, Istanbul, Turkey
[5] Department of Mathematics, University of Peshawar, Peshawar, Pakistan

ARTICLE INFO

Article history:
Received
Received in revised form
Accepted
Available online

Keywords:
Variational Model;
Fuzzy level set;
Intensity Inhomogeneity;
Active Contour;

ABSTRACT

Image segmentation is one of the crucial tasks in medical image processing and computer vision. The goal for image segmentation is to separate the pixels in the image into its constituent parts. Variational model in image segmentation involves formulating image segmentation as an optimization problem. The models seeking a partition of the image into meaningful regions by minimizing or maximizing an energy functional. These models often use geometric information to represent object boundaries. The models involve techniques such as active contours and level set methods. In this paper, the generalized mean model for image segmentation is investigated. The model is a 2D region-based model which utilizes the fuzzy level set method. The model is compared with the active contour without edges model also known as the Chan-Vese for three types of images: without noise, with noise and with sinusoidal intensity inhomogeneity. Based on the numerical results, the generalized mean model obtained higher accuracy and Dice similarity measure compared to the Chan-Vese model based on the tested images. The model is useful in medical imaging for disease detection, diagnosis, and treatment planning.

1. Introduction

Image segmentation aims to partition an image into several regions based on certain characteristics such as color, texture, and shapes. There exist several approaches for image segmentation, for example, edge detection and active contour models. Active contour models produce sub-regions with continuous boundaries while edge detection methods such as mSobel and Canny often produce sub-regions with discontinuous boundaries. In addition, edge detection methods highly depend on the threshold value. Active contour models usually used level set methods to obtain the segmented images. Level set methods are a broader class of techniques used for solving partial differential equations (PDEs) involving evolving curves and surfaces. These methods use a higher dimensional function, so-called the level set function to represent the evolving curve implicitly. The curve's position is determined by the zero level of the level set function. Level sets approaches to implement active contour models for image segmentation are preferable due to its flexibility and conveniences.

The Chan-Vese (CV) model by Chan and Vese [1] is a region-based active contour model that minimizes an energy function by segmenting images into two regions: foreground and background. The two regions are also known as two phase-based models. The CV model can be realized as a specific instance of a level set formulation for image segmentation. The functional consists of two terms: the fitting (data) term and the regularization term. The fitting term in the Chan-Vese model can be interpreted as the image-based forces that influence the evolution of the level set function. The term corresponds to image-based forces that attract the evolving contour towards object boundaries. Meanwhile the regularization term corresponds to curvature or smoothness terms that help maintain a continuous contour. The regularization term is also known as the length term. The convex formulation for the CV model is presented in Brown *et al.* [2].

The CV model assumes that the image intensities within each segmented region are constant. This assumption might not hold for images with intensity inhomogeneity where the intensity values vary across one object or features in the image. In addition, the model's performance is sensitive to the initial segmentation contour. Thus, incorrect initialization can lead to inaccurate segmentation results and converge to local minima. In addition, the regularization term in the model can lead to over-smoothing of object boundaries, which may cause loss of fine details. The performance of the CV model depends on the proper selection of the parameters, including those related to the data terms. Finding optimal parameters for different types of images can be challenging.

Ali *et al.* [3] proposed a generalized mean model where the arithmetic, geometric and harmonic mean are used as the data fitting term by tuning the parameter p in the generalized mean formula. The model in Ali *et al.* [3] is solved using zero level set function. The model is expanded to accommodate vector-valued images and multi-phase formulations. In Ali *et al.* [3], the authors evaluated their generalized mean (GM) model using a grayscale image contaminated with salt and pepper noise at a density of 0.01. Additionally, they applied Gaussian filtering to smooth the level set function. Meanwhile, Oh and Kwak [4] mentioned that the generalized mean of positive numbers can be written as a linear combination of the numbers. However, the application for the generalized mean functional in this paper is in the area of face reconstruction, clustering and object categorization.

To overcome the problems with the CV model, Rahman *et al.* [5] proposed a power mean model which extends the work in Ali *et al.* [3] to segment noisy images and images with outliers. The model utilizes the fuzzy level set to evolve the contour. Fuzzy level set methods combine the principles of fuzzy logic and level set techniques to enhance the robustness and flexibility of the traditional level set methods. Fuzzy logic allows for handling uncertainties and partial membership, which is particularly useful in scenarios where strict binary segmentation might not be appropriate. Instead of considering the level set function as strictly binary (inside or outside), fuzzy level set methods use fuzzy membership functions to represent the degree of membership of each point to a particular region. These membership values capture the uncertainty in the segmentation boundary.

The energy functional used in the level set PDEs in Rahman *et al.* [5] and Krinidis and Chatzis [6] are modified to include fuzzy membership values. This functional incorporates both image-based forces that drive the contour towards object boundaries and the fuzzy membership information. Fuzzy level set methods allow for smoother and more gradual transitions between segmented regions. According to Krinidis and Chatzis [6], fuzzy level set can help avoid sharp, unrealistic boundaries that traditional level set methods might produce. The fuzzy framework is presented in the joint image segmentation and registration of brain MRI with prior information in El-Melegy and Mokhtar [7]. In addition, Mondal [8] and Mondal *et al.* [9] introduced a resilient active contour method based on fuzzy energy, which incorporates information from both local and global energy components. Leveraging local information, including spatial distance and pixel intensity, helps manage issues arising from high intensity inhomogeneity and image noise. Furthermore, integrating global information is crucial to prevent adverse outcomes stemming from inadequate initialization.

In this paper, we review and investigate the model in Rahman *et al.* [5]. Based on three test images, we observed that the GM model works well without the length term in the functional. Thus, reducing the complexity of the model to determine the optimal pairs of the two parameters in the model: μ and p. The parameter p in the model plays an important role in obtaining the correct segmentation of the images. If the images are clean, any values of p work for segmenting the images. However, increasing the values of p will produce more accurate segmentation results such as improvement of the boundary detection at the corners of the objects.

The length term, also known as the perimeter term, is a component of the energy functional that encourages the contour to have a minimal length or perimeter. The length term contributes to the regularization of the contour shape. By minimizing the contour's length, the model encourages smoother, less convoluted contours. This helps prevent overly complex and jagged contour shapes that might lead to overfitting or inaccurate segmentation results. However, according to Zhang et al. 10], one can use a Gaussian filter to regularize the level set. The GM model works well without the length term and the Gaussian filter.

In addition, Krinidis and Chatzis [6] highlighted that when no noise is present in the image, we can use $\mu = 0$. The model manages to segment images with intensity inhomogeneity and produces global minimum. Compared to the CV model, the initial guess for the level set must vary to produce the optimal results. The parameter p acted like the smoothness term or the regularization parameters. Wu *et al.* [11] improved upon the previous study by Krinidis and Chatzis [6] by integrating a kernel metric capable of accurately detecting boundaries, particularly effective in images affected by noise, outliers, and low contrast.

Machine learning approaches, particularly deep learning, such as Li *et al.* [12], Badawy *et al.* [13], and Thanh *et al.* [14] have gained prominence in image segmentation due to their ability to learn complex patterns from data. Convolutional Neural Networks (CNNs) are commonly used for semantic and instance segmentation tasks. Machine learning-based image segmentation methods offer significant advantages, but the methods also come with certain disadvantages and challenges. First, the machine learning methods, especially deep learning, require large amounts of labeled training data. The process of acquiring and annotating such data can be time-consuming and expensive, particularly when the cost is the main issue in the specific problem-based applications. Second, deep learning models used for image segmentation can be complex. This complexity can hinder real-time or resource-constrained applications. Third, overfitting occurs when a model learns to memorize the training data instead of capturing meaningful patterns. This can lead to poor generalization of new and unseen data. Fourth, many machine learning algorithms have hyperparameters that need to be tuned to achieve optimal performance. Finding the right combination of hyperparameters can be a time-consuming process. Fifth, models trained on one dataset might not generalize well to other datasets with different characteristics. Domain adaptation or transfer learning might be needed to make models more versatile.

The structure of this paper is as follows: In the second section, an overview of the GM segmentation model and algorithm are presented. In the third section, the performance comparison for both models are made using two performance metrics on 2D real and synthetic medical images are presented. Conclusions of this paper are described in the last section.

2. Methodology

The generalized mean, also known as the power mean or Hölder mean, is a mathematical concept used to calculate a single value that represents the "average" or "central tendency" of a set of numbers. It extends the idea of the arithmetic mean to incorporate a parameter denoted as p, which allows for different weighting of the individual numbers in the set. The formula for the generalized mean for a continuous function $f(\boldsymbol{x})$ over an interval $[a, b]$ is as follows:

$$M_p = \left(\frac{1}{b-a}\int_a^b f(\boldsymbol{x})^p d\boldsymbol{x}\right)^{\frac{1}{p}}.$$

For a monotonic increasing function $f(\boldsymbol{x})$, the functional for the generalized mean can be approximated using:

$$M_p = \int \left(f(\boldsymbol{x})^2\right)^p d\boldsymbol{x}.$$

2.1 Two Phase Generalized Mean (GM) Model for Image Segmentation

Defined an image I on $\Omega \subset \Re^2$, and $\Omega_i \subseteq \Omega$ are disjoint connected open subsets with a piecewise smooth boundary C where $I(\boldsymbol{x}) > 0$ is the intensity value at a certain pixel in which $\boldsymbol{x} = (x_1, x_2)$. The minimization functional for GM model in Rahman *et al.* [5] is expressed as

$J(\emptyset(\boldsymbol{x}), c_1, c_2) = \mu \int_\Omega \lvert\nabla\emptyset(\boldsymbol{x})\rvert d\boldsymbol{x}$ $\int_\Omega \alpha(\boldsymbol{x})(I(\boldsymbol{x}) - c_1)^2[\emptyset(\boldsymbol{x})]^2 d\boldsymbol{x} +$ $J(\emptyset(\boldsymbol{x}), c_1, c_2) = \mu \int_\Omega \lvert\nabla\emptyset(\boldsymbol{x})\rvert d\boldsymbol{x}$ $\int_\Omega \beta(\boldsymbol{x})(I(\boldsymbol{x}) - c_2)^2[1 - \emptyset(\boldsymbol{x})]^2 d\boldsymbol{x}$	(1)

$$J(\emptyset(\boldsymbol{x}), c_1, c_2) = \mu \int_\Omega \lvert\nabla\emptyset(\boldsymbol{x})\rvert d\boldsymbol{x} + \int_\Omega \alpha(\boldsymbol{x})(I(\boldsymbol{x}) - c_1)^2[\emptyset(\boldsymbol{x})]^2 d\boldsymbol{x} + \int_\Omega \beta(\boldsymbol{x})(I(\boldsymbol{x}) - c_2)^2[1 - \emptyset(\boldsymbol{x})]^2 d\boldsymbol{x} \quad (1)$$

where

$$\alpha(\boldsymbol{x}) = ((I(\boldsymbol{x}) - c_1)^2[\emptyset(\boldsymbol{x})]^2)^{p-1}$$
$$\beta(\boldsymbol{x}) = ((I(\boldsymbol{x}) - c_2)^2[1 - \emptyset(\boldsymbol{x})]^2)^{p-1} \quad (2)$$

The first functional in (1) is the length term to regularize the curve *C*. However, when there is noise in the image, we need $\mu \neq 0$ or the Gaussian filter to minimize the length of *C*. The length term is also known as the total variation term. To maximize the detection of objects, regardless of their size, it's preferable to keep the parameter μ at a small value. Conversely, when the focus is on identifying larger objects or excluding smaller ones, it's advantageous to set μ to a larger value. Thus, μ serves as the scaling parameter. When there is no noise in the image, the value of μ can be set to zero. Lower value of μ caused crookedness and less smoothing of the boundaries between the foreground and the background in the images. The boundaries appeared irregular and jagged. However, the lower value of μ managed to capture finer details in the images. Meanwhile, for the larger value of μ, the model produced smoother boundaries. In addition, with large μ, the fine details in the images will also smooth out.

The fuzzy membership function, $\emptyset(\boldsymbol{x})$ are defined as follow:

$$\begin{aligned} C &= \{(x_1, x_2) \in \Omega : \emptyset(\boldsymbol{x}) = 0.5\}, \\ \text{inside } (C) &= \{(x_1, x_2) \in \Omega : \emptyset(\boldsymbol{x}) > 0.5\}, \\ \text{outside } (C) &= \{(x_1, x_2) \in \Omega : \emptyset(\boldsymbol{x}) < 0.5\} \end{aligned} \quad (3)$$

and

$$c_1 = \frac{\int_\Omega \alpha(\boldsymbol{x}) I(\boldsymbol{x})[\emptyset(\boldsymbol{x})]^2 d\boldsymbol{x}}{\int_\Omega \alpha(\boldsymbol{x})[\emptyset(\boldsymbol{x})]^2 d\boldsymbol{x}}, \qquad c_2 = \frac{\int_\Omega \beta(\boldsymbol{x})\, I(\boldsymbol{x})\,[1-\emptyset(\boldsymbol{x})]^2 d\boldsymbol{x}}{\int_\Omega \beta(\boldsymbol{x})\,[1-\emptyset(\boldsymbol{x})]^2 d\boldsymbol{x}} \tag{4}$$

The original function for the generalized mean is given by

$$J(\emptyset(\boldsymbol{x}), c_1, c_2) = \int_\Omega [(I(\boldsymbol{x}) - c_1)^2[\emptyset(\boldsymbol{x})]^2]^p d\boldsymbol{x} + \int_\Omega [(I(\boldsymbol{x}) - c_2)^2[1-\emptyset(\boldsymbol{x})]^2]^p d\boldsymbol{x} \tag{5}$$

According to Oh and Kwak [4], the generalized mean of a set of positive numbers can be represented as a non-negative linear combination of its elements. Thus, the functional in (5) is reduced to the functional in (1). By keeping c_1 and c_2 fixed in (4), then minimized $J(\emptyset(\boldsymbol{x}), c_1, c_2)$ with respect to $\emptyset$ and $\mu = 0$, it becomes

$$\emptyset(\boldsymbol{x}) = \frac{1}{1 + \left(\frac{\alpha(\boldsymbol{x})(I(\boldsymbol{x}) - c_1)^2}{\beta(\boldsymbol{x})(I(\boldsymbol{x}) - c_2)^2}\right)} \tag{6}$$

Krinidis and Chatzis [6] noted that in two-phase image segmentation, employing the fuzzy energy excluding the length term and utilizing the Jacobi iteration method results in convergence after precisely one sweep, regardless of the initial model condition.

Based on Rahman *et al.* [5], the updated value of $\emptyset(\boldsymbol{x})$ is used in the Euler-Lagrange equation obtained by minimized $J(\emptyset(\boldsymbol{x}), c_1, c_2)$ with respect to $\emptyset$ and $\mu \neq 0$. The Euler-Lagrange equation is solved by introducing τ as an artificial time and using fully explicit scheme. The GM segmentation model is summarized in **Algorithm 1**.

Algorithm 1: GM Segmentation Model

BEGIN

Input: Image, *I*

Output: The segmentation of image, $C{:}\, Seg(I(\boldsymbol{x}))$

1. Initialization:

   $$I, p, \mu, maxit, \emptyset(\boldsymbol{x})$$

2. **For** $iter = 1, \ldots\ldots, maxit$
   (a) Update $\alpha(\boldsymbol{x})$ and $\beta(\boldsymbol{x})$ using (2).
   (b) Update the value of c_1 and c_2 using (4).
   (c) Update $\emptyset(\boldsymbol{x})$ using (6).
   (d) Smooth $\emptyset(\boldsymbol{x})$ using 2D Gaussian convolution function.
3. End **For**.
4. Compute the segmentation of the image using $\emptyset(\boldsymbol{x})$.

END

3. Results

In this section, the numerical results for the GM model are compared with the CV model using the build in MATLAB function:

```
B = activecontour(I,mask,500,'Chan-Vese','SmoothFactor',mu_cv)
```

where `mask` is the initial segmentation of the image and `mu_cv` is the regularization parameters for the length term in the functional. 500 is the maximum number of iterations. To measure the achievement of the models, the accuracy and Dice similarity measures are calculated as follows:

$$a = \frac{TP + TN}{FN + FP + TP + TN}$$

and

$$d = \frac{2TP + TN}{2TP + FP + FN}$$

where
- TP : True positive
- FP : False positive
- TN : True negative
- FN : False negative

We used the following MATLAB command:

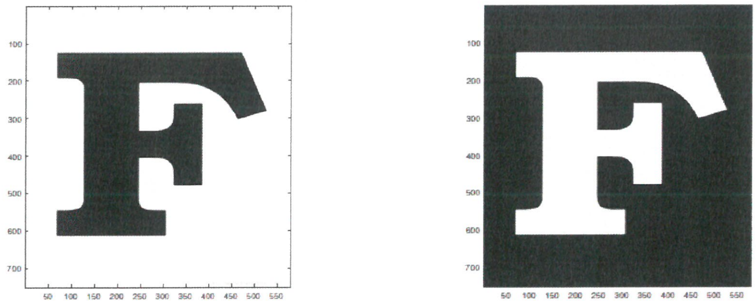
```
[a,~,~,~,~,d]=EvaluateImageSegmentationScores(A,B);
```

where `A,B` is the ground truth and the segmented results from the two models respectively. The values for a and d are $0 \leq a, d \leq 1$. The smaller values indicate the models are less accurate and has lower similarity with the ground truth. Thus, we aim for *a* and *d* close to or equal to 1. We used μ=0 in (1) for all experiments and varies *p* to obtain the highest values of *a* and *d*.

Table 1
Values of the accuracy and Dice similarity measures for Test 1,2 and 3

	Test 1		Test 2		Test 3	
Model	*a*	*d*	*a*	*d*	*a*	*d*
GM *p*=0.5	1.0000	1.000	0.7515	0.4841	0.8650	0.7446
GM *p*=1.0	1.0000	1.000	0.9993	0.9971	0.9158	0.8581
CV μ_cv=0	0.9974	0.9952	0.9978	0.9906	0.8567	0.7240
CV μ_cv=0.1	0.9978	0.9958	0.9986	0.9940	0.8550	0.7272

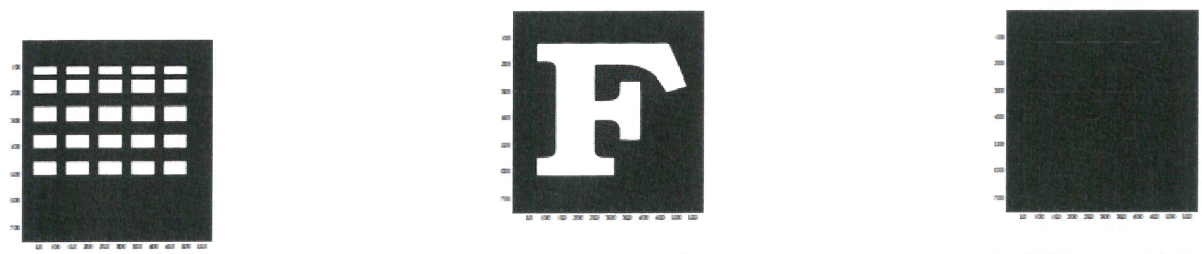
3.1 Test 1: Image without Noise



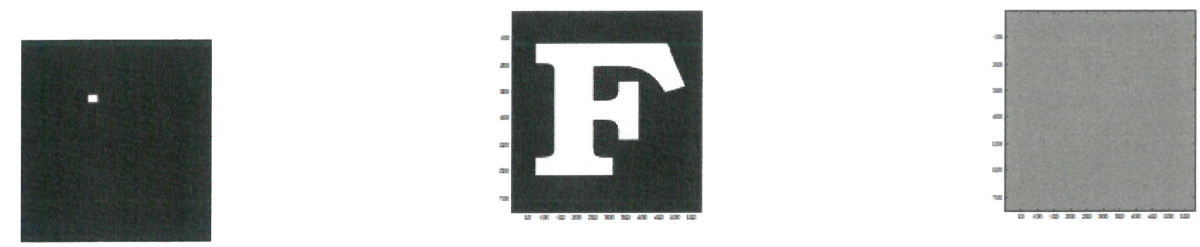
(a) Image I (b) Seg(I)

Fig. 1. The image in (a) contains no noise and the ground truth segmentation of the image is given in (b)

In the first experiment, we tested the models using the letter F. The image has no noise and consists of several sharp corners. The size of the image is 751×580 pixels. The results for CV and GM models are shown in Figure 2. Different initial mask for the CV model will produce different segmentation results. The optimal initial mask is given in Figure 2 (a) using μ = 0.1. The second row in Figure 2 shows the results for GM model. The image in Test 1 has well-defined object boundaries and zero intensity variations. Choosing an appropriate initial contour is crucial to guide the segmentation process. The CV model is non-convex image segmentation model. Thus, segmentation of the image depends on the initialization or the initial state of the active contour. The model used initialization to evolve and segment the image. Thus, we must provide a good initialization which is close to the object boundaries. The CV model has parameters that influence the segmentation result: the weight of the region fitting term and the regularization term μ. It's important to note that even though the image in Test 1 is clean and noise-free, the success of the CV model depends on the object's contrast, shape complexity, and the choice of parameters. We had to experiment with different initialization and value of μ to achieve the best segmentation result as shown in Figure 2 (b). From Table 1, increasing μ from 0 to 0.1 in the CV model improved the values of *a* and *d*. However, the largest values of *a* and *d* are given by the GM model using *p* = 0.5 and 1.0. From this experiment, the GM model outperform the CV model.



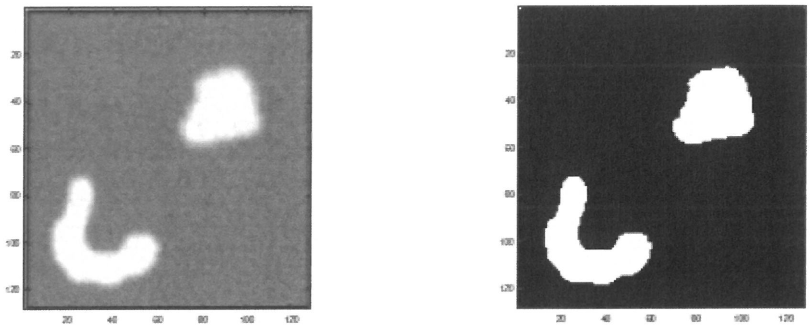
(a) CV initial mask (b) $Seg(I)$ using CV model (c) Difference with the ground truth



(d) GM initial mask (e) $Seg(I)$ using GM model (f) Difference with the ground truth

Fig. 2. Results for Test 1 using the GM and CV models

3.2 Test 2: Image with Low Level of Noise

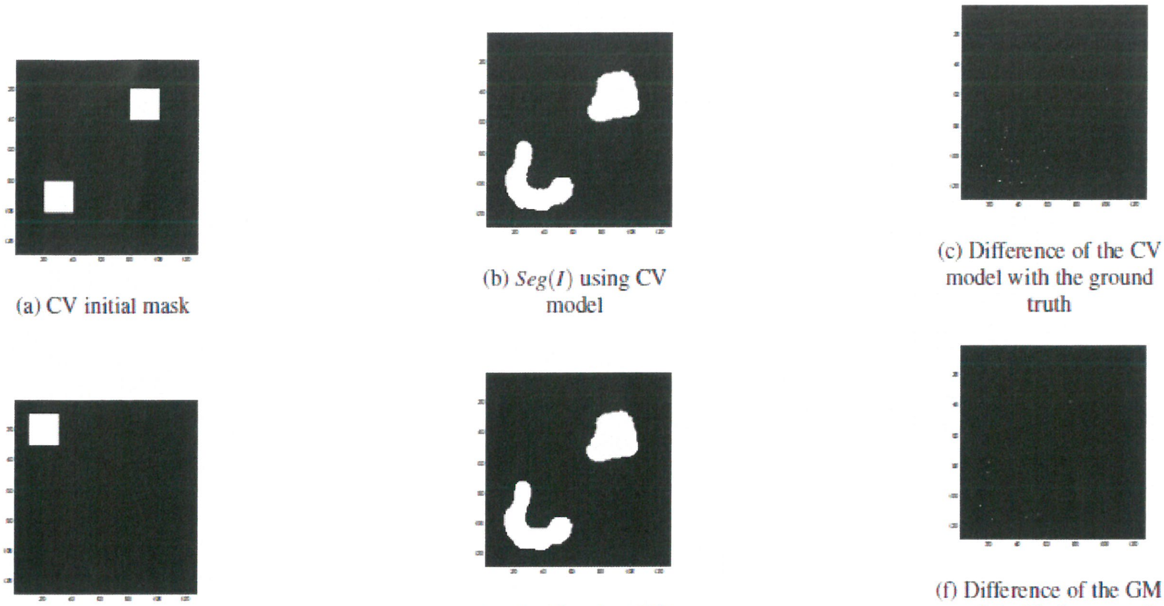


(a) Image I (b) Seg(I)

Fig. 3. The image in (a) contains noise and the ground truth segmentation of the image is given in (b)

In the second experiment, the models are tested for image with noise. The images are from [15] with size 128×128. Image segmentation in the presence of noise can be challenging, as noise can introduce errors and affect the accuracy of segmentation algorithms. There exists several image denoising methods such as the Gaussian, median, or bilateral filters which aim to reduce noise while preserving edges in the images. These filters can help create a smoother input image for segmentation. However, in Test 2, we are not using any image denoising methods. The image in Test 2 contains two objects and low level of noise. Similarly, as in Test 1, the initial mask for the CV model must be chosen close to the object boundaries. If the initial mask contains only one square box, then the segmentation results will only produce one object. Thus, we must use two square boxes as the initialization to the CV model.

The results for the CV and GM models as shown in Figure 4. The corresponding values of a and d are shown in Table 1. Higher values of a and d are obtained from the GM model using $p = 1$. When $p = 1$, the generalize mean is equivalence to the arithmetic mean. The performance of the CV and GM models are affected when noise is present in the image. Noise introduces variations in pixel intensities, which can lead to challenges in accurately segmenting objects using intensity-based methods like the CV and GM models. The GM model works only for $p \geq 0.6$ with the presence of noise. When $p < 0.6$, the GM model produce inaccurate boundaries of the two objects. Noise led to local intensity fluctuations that do not correspond to actual object boundaries. This causes the GM model to produce inaccurate segmentation of the objects for $p < 0.6$.



(a) CV initial mask

(b) $Seg(I)$ using CV model

(c) Difference of the CV model with the ground truth

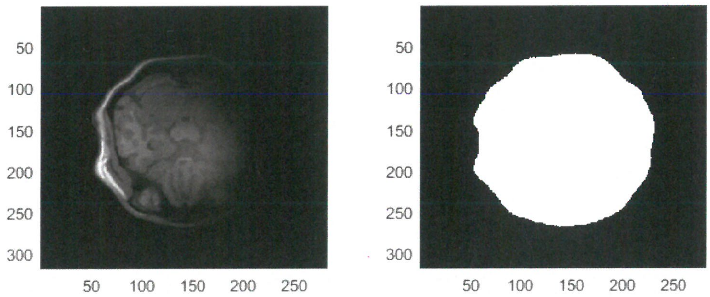
(d) GM initial mask

(e) $Seg(I)$ using GM model

(f) Difference of the GM model with the ground truth

Fig. 4. Results for Test 2 using the GM and CV models

3.3 Test 3: Image with Sinusoidal Inhomogeneity

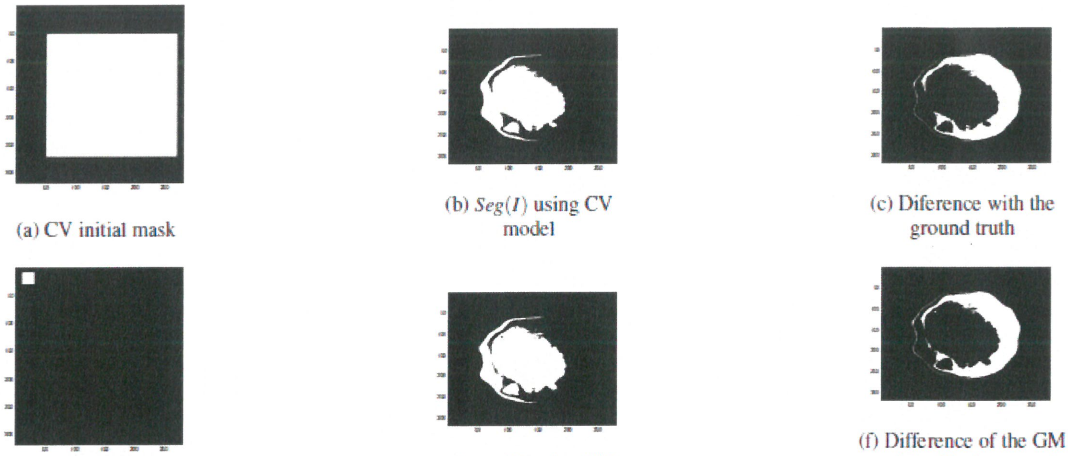


(a) Image I

(b) Seg(I)

Fig. 5. Images for Test 3 where the image in (a) contains sinusoidal inhomogeneity and the ground truth segmentation is given in (b)

We used image from [16] to test the two models for images with intensity inhomogeneity. Intensity inhomogeneity, also known as intensity bias or shading, refers to variations in image



(a) CV initial mask (b) $Seg(I)$ using CV model (c) Diference with the ground truth

(d) GM initial mask (e) $Seg(I)$ using GM model (f) Difference of the GM model with the ground truth.

Fig. 6. Results for Test 3 using the GM and CV models

intensities that occur due to uneven illumination, sensor artifacts, or other factors. It can significantly affect image analysis and computer vision tasks, such as segmentation, object detection, and feature extraction. Understanding and correcting intensity inhomogeneity is crucial for obtaining accurate and reliable results in these tasks. Imperfections in camera sensors, such as vignetting, can result in spatially varying response to light, leading to intensity variations. Uneven illumination can cause objects to appear brighter or darker in different regions, making it difficult for segmentation algorithms to accurately distinguish between object and background. Inhomogeneous intensity can affect the accuracy of feature extraction methods that rely on local intensity patterns or gradients. The performance of the CV model can be negatively impacted when there is intensity inhomogeneity (uneven illumination) present in the image. Intensity inhomogeneity causes variations in pixel intensities that are not related to object boundaries, and this can pose challenges for segmentation methods that rely heavily on intensity-based information, such as the CV and GM models.

Based on the values of a and d in Table 1, we observed that the CV model is at disadvantages when the image contains sinusoidal inhomogeneity as reported in Ali *et al.* [17]. We used sinusoidal signal with amplitude 50, frequency=0.001 and zero phase to produce image in Figure (5) (a). It is still a challenge for both models to produce a prefect segmentation results for Test 3.

4. Conclusions

In this paper, we reviewed and investigated the GM model for image segmentation and showed that the model can work for μ=0. The parameter p acted similarly to the regularization parameter. However, if the images contain noise, choosing the optimal value of p is crucial. We compared the model with the state-of-the-art model which is the CV model using the build -in MATLAB command. We performed three set of experiments and compared the values of accuracy and dice metric. The GM model produced higher values of accuracy and dice metric for three datasets. If the images consist of objects with intensity inhomogeneity due to bias field or during the image acquisition process, the CV model is at disadvantages. For the same object with significant different intensity values, the model will either segment the darker or brighter regions of the object.

Conflict of interest

All authors are requested to disclose any actual or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.

Acknowledgement

The research and writing of this work are funded by Ministry of Higher Education of Malaysia and National Defence University of Malaysia under grants RACER/1/2019/ICT01/UPNM/1 and UPNM/2022/GPPP/SG/15.

References

[1] Chan, Tony, and Luminita Vese. "An active contour model without edges." In International conference on scale-space theories in computer vision, pp. 141-151. Berlin, Heidelberg: Springer Berlin Heidelberg, 1999. https://doi.org/10.1007/3-540-48236-9_13

[2] Brown, Ethan S., Tony F. Chan, and Xavier Bresson. "Completely convex formulation of the Chan-Vese image segmentation model." International journal of computer vision 98 (2012): 103-121. https://doi.org/10.1007/s11263-011-0499-y

[3] Ali, Haider, Amna Shujjahuddin, and Lavdie Rada. "A new active contours image segmentation model driven by generalized mean with outlier restoration achievements." International Journal of Pattern Recognition and Artificial Intelligence 34, no. 11 (2020): 2054026. https://doi.org/10.1142/S0218001420540269

[4] Oh, Jiyong, and Nojun Kwak. "Generalized mean for robust principal component analysis." Pattern Recognition 54 (2016): 116-127.https://doi.org/10.1016/j.patcog.2016.01.002

[5] Rahman, Afzal, Haider Ali, Noor Badshah, Muhammad Zakarya, Hameed Hussain, Izaz Ur Rahman, Aftab Ahmed, and Muhammad Haleem. "Power mean based image segmentation in the presence of noise." Scientific Reports 12, no. 1 (2022): 21177. https://doi.org/10.1038/s41598-022-25250-x

[6] Krinidis, Stelios, and Vassilios Chatzis. "Fuzzy energy-based active contours." IEEE Transactions on Image Processing 18, no. 12 (2009): 2747-2755. https://doi.org/10.1109/tip.2009.2030468

[7] El-Melegy, Moumen, and Hashim Mokhtar. "Fuzzy framework for joint segmentation and registration of brain MRI with prior information." In The 2010 International Conference on Computer Engineering & Systems, pp. 9-14. IEEE, 2010. https://doi.org/10.1109/icces.2010.5674904

[8] Mondal, Ajoy. "Fuzzy energy based active contour model for multi-region image segmentation." Multimedia Tools and Applications 79 (2020): 1535-1554. https://doi.org/10.1007/s11042-019-08207-7

[9] Mondal, Ajoy, K. Ramachandra Murthy, Ashish Ghosh, and Susmita Ghosh. "Robust image segmentation using global and local fuzzy energy based active contour." In 2016 IEEE international conference on fuzzy systems (FUZZ-IEEE), pp. 1341-1348. IEEE, 2016. https://doi.org/10.1109/FUZZ-IEEE.2016.7737845

[10] K. Zhang, H. Song, and L. Zhang, "Active contours driven by local image fitting energy," Pattern recognition 43, 1199-1206 (2010). https://doi.org/10.1016/j.patcog.2009.10.010

[11] Wu, Yue, Wenping Ma, Maoguo Gong, Hao Li, and Licheng Jiao. "Novel fuzzy active contour model with kernel metric for image segmentation." Applied Soft Computing 34 (2015): 301-311. https://doi.org/10.1016/j.asoc.2015.04.058

[12] Y. Li, H. Gu, H.Wang, P. Qin, and J.Wang, "Busnet: A deep learning model of breast tumor lesion detection for ultrasound images," Frontiers in Oncology 12, 848271 (2022).

[13] S. M. Badawy, A. E.-N. A. Mohamed, A. A. Hefnawy, H. E. Zidan, M. T. GadAllah, and G. M. El-Banby, "Automatic semantic segmentation of breast tumors in ultrasound images based on combining fuzzy logic and deep learning-a feasibility study," PloS one 16, e0251899 (2021). https://doi.org/10.1371/journal.pone.0251899

[14] D. N. Thanh, N. H. Hai, P. Tiwari, V. S. Prasath, et al., "Skin lesion segmentation method for dermoscopic images with convolutional neural networks and semantic segmentation," 45, 122-129 (2021).

[15] M. Ibrahim, Variational models and numerical algorithms for effective image registration, Ph.D. thesis, University of Liverpool (2015).

[16] A. Myronenko and X. Song, "Intensity-based image registration by minimizing residual complexity," IEEE transactions on medical imaging,29, 1882-1891 (2010).

[17] Ali, Haider, Lavdie Rada, and Noor Badshah. "Image segmentation for intensity inhomogeneity in presence of high noise." IEEE Transactions on Image Processing 27, no. 8 (2018): 3729-3738.