EFFECTS OF NON-NEWTONIAN HYBRID NANOFLUID HEAT TRANSFER AND ENTROPY GENERATION OVER A HORIZONTAL SHRINKING SURFACE

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MASTER OF SCIENCE
(MATHEMATICS)

UNIVERSITI PERTAHANAN NASIONAL MALAYSIA

2024

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Thesis submitted to the Centre for Graduate Studies, Universiti Pertahanan Nasional Malaysia, in fulfilment of the requirements for the Degree of Master of Science (Mathematics)

2024

ABSTRACT

The heat transport and entropy generation of Magnetohydrodynamic (MHD) non-Newtonian Powell-Eyring hybrid nanofluid near the stagnation point over a horizontal shrinking surface is examined using Tiwari and Das model, with graphene oxide (GO) as the main nanomaterial. Three problems of boundary layer flow are assessed, where GO is combined with iron dioxide (Fe_2O_4) and ethylene glycol ($C_2H_6O_2$), molybdenum disulfide (MoS_2) and glycerine ($C_3H_8O_3$), and lastly molybdenum disulfide (MoS_2) and ethylene glycol ($C_2H_6O_2$). The first, second, and third problems study Joule heating, slips, and radiation effects, respectively. The mathematical modelling for each problem consists of continuity, momentum, and energy equations in partial differential equations (PDEs). Using suitable similarity transformations, the PDEs are then reduced to ordinary differential equations (ODEs). The ODEs are solved numerically by utilizing the bvp4c, a built-in solver in MATLAB software. The numerical results are illustrated in the form of figures and tables, where they display the velocity profile, temperature profile, skin friction, Nusselt number and entropy generation. The numerical results presented are gained by varying the value of several governing variables such as magnetic field, radiation, rapidity slip, thermal slip, heat source, viscous dissipation, Joule heating, Biot number and suction. The findings obtained reveal that the augmentation of GO concentration ameliorates the temperature of the fluid while depleting the rapidity of fluid, rate of heat transport and production of entropy. The amplification of thermal radiation intensifies the temperature of the liquid and plunges the formation of entropy. The solutions for the shrinking surface are found to be non-unique or known as dual solutions. Stability analysis is conducted by introducing disturbance to check the steadiness of both

solutions. The stability analysis showed that the upper branch solution fulfils the characteristics of a stable solution. Hence, the lower branch solution is regarded as an unsteady solution.

ABSTRAK

Pemindahan haba dan penghasilan entropi bagi aliran Magnetohidrodinamik (MHD) dalam nanobendalir hibrid Powell-Eyring yang berdekatan dengan titik genangan dikaji di atas permukaan melintang yang mengecut menggunakan model Tiwari dan Das, dengan grafen oksida (GO) sebagai nanozarah utama. Tiga masalah aliran lapisan sempadan yang berbeza dikaji iaitu, GO digabungkan dengan ferum oksida (Fe_2O_4) dan etilena glikol ($C_2H_6O_2$), molibdenum disulfida (MoS_2) dan gliserin ($C_3H_8O_3$), dan yang terakhir molibdenum disulfida (MoS_2) dan etilena glikol ($C_2H_6O_2$). Permasalahan pertama, kedua, dan ketiga mengkaji pemanasan Joule, kesan gelinciran dan radiasi terma. Model matematik bagi semua masalah aliran lapisan sempadan dalam tesis ini mengandungi persamaan keselanjaran, momentum dan tenaga dalam bentuk persamaan pembezaan separa (PDEs). Persamaan pembezaan separa tersebut kemudian digubah menjadi persamaan pembezaan biasa (ODEs) menggunakan penjelmaan keserupaan sepadan. ODEs kemudian diselesaikan secara berangka menggunakan fungsi bvp4c yang terbina dalam perisian MATLAB. Keputusan berangka ditunjukkan dalam bentuk graf dan jadual, yang memaparkan profil halaju, profil suhu, pekali geseran kulit, nombor Nusselt dan penjanaan entropi. Keputusan berangka diperolehi dengan memvariasikan nilai-nilai bagi beberapa pemboleh ubah seperti medan magnet, radiasi, gelincir halaju, gelincir terma, penjanaan haba, pelesapan likat, pemanasan Joule, nombor Biot dan sedutan. Dapatan kajian mendedahkan bahawa penambahan kepekatan GO menaikkan suhu bendalir, dalam masa sama mengurangkan kelajuan cecair, kadar transportasi haba dan penjanaan entropi sistem. Apabila nilai radiasi dinaikkan, suhu bendalir akan bertambah kuat dan penghasilan entropi akan menjunam. Solusi-solusi bagi

permukaan yang mengecut didapati menghasilkan solusi yang tidak unik atau dikenali sebagai penyelesaian dual. Analisis kestabilan dijalankan dengan memperkenalkan gangguan untuk menyemak kemantapan kedua-dua penyelesaian. Daripada analisis tersebut, penyelesaian pertama didapati memenuhi karakter yang diperlukan untuk penyelesaian yang stabil. Oleh itu, penyelesaian kedua dianggap sebagai penyelesaian yang tidak stabil.

ACKNOWLEDGEMENTS

In the name of Allah, most gracious and most merciful,

First and foremost, I would like to acknowledge the support, knowledge, advice and encouragement that I received from my main supervisor, Dr Nor Ain Azeany binti Mohd Nasir. Not to forget the support from my co-supervisor Dr Mohd Anuar bin Jamaludin.

Special thanks to National Defence University of Malaysia and Ministry of Higher Education Malaysia for the funds received in supporting this research, especially for publication of papers and attendance to conferences.

A big appreciation for my seniors and friends for all kinds of assistance offered during the process to complete this journey. Not to forget my parents, siblings, and family members. Words cannot express how much I appreciate all the kindness that I received while working on this thesis.

APPROVAL

The Examination Committee has met on **5 January 2024** to conduct the final examination of **Nur Aisyah binti Aminuddin** on her degree thesis entitled **‘Effects of Non-Newtonian Hybrid Nanofluid Heat Transfer and Entropy Generation Over a Horizontal Shrinking Surface’.**

The committee recommends that the student be awarded the of Master of Science (Mathematics).

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APPROVAL

This thesis was submitted to the Senate of Universiti Pertahanan Nasional Malaysia and has been accepted as fulfilment of the requirements for the degree of **Master of Science (Mathematics)**. The members of the Supervisory Committee were as follows.

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Academic session : 2022/2023

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LIST OF ABBREVIATIONS

AuNP	-	Gold nanoparticle
$C_2H_6O_2$	-	Ethylene Glycol
$C_3H_8O_3$	-	Glycerine
CNT	-	Carbon nanotube
CuNP	-	Copper nanoparticle
EMHD	-	Electromagnetohydrodynamic
Fe_2O_4	-	Iron Dioxide
GO	-	Graphene Oxide
H_2O	-	Water
MHD	-	Magnetohydrodynamic
MoS_2	-	Molybdenum Disulfide
ODE	-	Ordinary Differential Equation
PDE	-	Partial Differential Equation
PV	-	Photovoltaic

LIST OF SYMBOLS

a, b	-	Constant
B_0	-	Strength of magnetic field
$\mathrm{B_r}$	-	Brinkmann Number
Bi	-	Biot Number
C_f	-	Skin Friction Coefficient
C_p	-	Specific Heat Capacity
D	-	Dimensional Temperature Jump Parameter
Ec	-	Eckert Number
E_G	-	Entropy Generation
G	-	Hartman Number
h	-	Convection Heat Transfer Coefficient
H	-	Thermal Slip
I	-	Identity Tensor
j_0	-	Applied Current Density
k	-	Thermal Conductivity
k^*	-	Absorption Parameter
L	-	Length Of Surface
m_0	-	Characteristic Magnetisation
M	-	Magnetic Parameter
N_G	-	Dimensionless Entropy Generation
Nu	-	Nusselt Number
Nu_x	-	Local Nusselt Number
p	-	Pressure
Pr	-	Prandtl Number
Q	-	Heat Absorption/Generation Constant
Q_0	-	Heat Absorption/Generation Parameter
q_r	-	Radiation Heat Flux
q_w	-	Heat Flux
U_w	-	Surface Velocity

Rd	-	Radiation Parameter
Re	-	Reynolds Number
S	-	Suction/Injection
t	-	Time
T	-	Temperature of Fluid
T_w	-	Surface Temperature
T_∞	-	Ambient Temperature
u, v	-	Velocity Components Along x-Axis and y-Axis
u_e	-	Free stream velocity
V	-	Vector of Hybrid Nanofluid Velocity
x, y	-	Cartesian Coordinates
		Subscripts
c	-	Critical Value
$s1, s2$	-	Particles
hnf	-	Hybrid Nanofluid
f	-	Base Fluid
		Symbols
∇	-	Laplace Operator
Δ	-	Dimensionless Parameter
		Greek Symbols
α	-	Stretching/Shrinking Parameter
β^*, ε	-	Fluid Parameters of Powell-Eyring
γ	-	Eigenvalue
δ	-	Thickness of Boundary Layer
ϵ	-	Velocity Slip
ϵ_0	-	Dimensional Velocity Slip Parameter
η, ψ, ϑ	-	Similarity Transformations
μ	-	Dynamic Viscosity
ν	-	Kinematic Viscosity
ξ	-	Heat Diffusivity
ρ	-	Density
ρC_p	-	Heat Capacity
σ	-	Electric Conductivity

σ^*	-	Stefan Constant
ς, χ	-	Material Parameters
τ	-	Dimensionless Time Variable
τ_c	-	Cauchy Stress Tensor
τ_{ij}	-	Stress Tensor
τ_w	-	Wall Shear Stress
ϕ	-	Concentration
ϕ_a	-	Ratio of Dynamic Viscosity for Hybrid Nanofluid
ϕ_b	-	Ratio of Density for Hybrid Nanofluid
ϕ_c	-	Ratio of Heat Capacity for Hybrid Nanofluid
ϕ_d	-	Ratio of Thermal Conductivity for Hybrid Nanofluid
ϕ_e	-	Ratio of Electrical Conductivity for Hybrid Nanofluid
ω	-	Width of Electrodes

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CHAPTER 1

INTRODUCTION

1.1 Heat Transfer

Essentially, the motion of energy from one area to another due to the existence of temperature difference is outlined as heat transfer. The roles of heat transfer are either to discard the heat from a system or to seal off the heat in a system. The ability of a device to cool or heat enormously depends on the speed and amount of heat transported. Liquids can aid in transmitting heat between surfaces and fluid in machines when there is imbalance of heat. Hence, it is exceptionally vital to investigate ways to upgrade the thermal properties of liquids to boost their heat transfer capability. Countless scholars have explored the thermal properties of fluids by manipulating the type of fluids, surfaces, and external sources of effects. This research area is favoured by many owing to its significance in plenty of tools and appliances, including the solar power system, aeroplanes, nuclear devices, and oil and gas sector. Basically, heat transfer can be attained by a few means namely conduction, convection, and radiation.

1.1.1 Conduction

In conduction, there is no motion of the fluid. The dissimilarity of temperature in two different spaces leads the heat to transmit from a heated area to a chilled area. In the long run, thermal equilibrium will be reached as the difference in temperature diminishes.

1.1.2 Convection

Unlike conduction, an immense motion in convection leads to the shifting of heat. It is compulsory for convection to have advection and diffusion. Advection is the heat conveyed by the huge motion in the liquid, while diffusion is the Brownian motion of the particles in the liquid. Convection can only occur across liquids and gases or between solid and liquid. Two types of convection are natural convection and forced convection. Natural convection arises when there is a density difference in the heated fluid. Forced convection is regulated by external factors that are located outside of the system, such as pumps. It does not occur due to the heated fluid. Instead of natural convection, a higher rate of thermal transmission is more doable in forced convection.

1.1.3 Radiation

In radiation, nothing is swapped, and no medium is required. When the particles vibrate, it creates electromagnetic waves that transmit energy through radiation. The radiation energy does not only emerge from the surface but the entire spot of the body. Temperature gradient is absent in radiation. Thus, an object close to the source still gets to experience the heat.

1.2 Non-Newtonian Fluid

Newtonian fluids have uniform and unchanged viscosity, while non-Newtonian fluid's viscosity is not consistent and will change when a dissimilar size of force is applied. Principally, non-Newtonian fluid does not stick to Newton's law of viscosity. The non-linear manner of non-Newtonian fluid is impracticable to be solved analytically, and this type of fluid often has power law relationship. One of the traits of non-Newtonian fluid is that shear stress is nonlinear to the shear rate. This relationship is known as the velocity gradient. Blood, paints, shampoos, oils, polymers, starches, and dyes are examples of non-Newtonian fluids. Toothpaste can be used to explain the viscous behaviour of non-Newtonian liquid. When the cap is opened, the toothpaste will not come out even if the tube is upside down. However, only when we apply force to the tube the toothpaste will flow out and acts as a liquid. This proved the inconsistency of viscosity in non-Newtonian fluid.

Newtonian fluid can be explained using a single reference equation. However, it is not the case for non-Newtonian fluid because of its rheological features. These rheological features constitute of stress, viscosity and so on, and can only be figured out using constitutive equations. Constitutive equations are compulsory to interpret the distinct connection between the stress and shear rates of non-Newtonian liquids. These tangled properties require an advanced fluid model to inspect the fluid flow rather than the basic Navier-Stokes equations. On that account, a lot of scholars have established several non-Newtonian models with diverse thermophysical details of this type of