A MATHEMATICAL MODEL DEVELOPMENT FOR THE QUASI-STATIC LATERAL COLLAPSE OF THE GENERALISED GEOMETRIC HOLLOW SHAPES

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DEDICATION

"Especially for

My beloved mother and father,

Maimun binti Abdullah Munir

and Kamardan bin Sati

My beloved Wife

Noor Hidayah binti Mat Sari

My beloved Childeren

Nurdina Syaherah,

Muhammad Danish Syazwan

and Ahmad Daniyal Syafie

Thank you very much for all your prayers for me.

May Allah bless you all with a lot of goodness.

ABSTRACT

The purpose of this research is to develop a general predictive **mathematical** model of the deformation behaviours for various symmetric geometrical tubes under lateral compression between two flat rigid plates. The mathematical model has been proposed based on rigid, perfectly plastic model and the energy balance method. The mathematical models are divided into two cases i.e. 'Case 1' and 'Case 2' based on the geometrical shapes of the tubes. ‘Case 1’ is for shapes with number of sides 6, 10, 14 and so on such as hexagonal, decagonal and tetra-decagonal tubes. Whereas, ‘Case 2’ is for shapes with number of sides 4, 8, 12 and so on such as square, octagonal and dodecagonal tubes. The prediction or assumption used in this mathematical model was that the tubes would deform in phase by phase during plastic deformation. In order to achieve this purpose, the deformation behaviour and the energy-absorption performance of various geometrical tube shapes need to be determined. The geometrical tubes shapes which were studied include square, hexagonal, octagonal, decagonal, dodecagonal and tetra-decagonal tubes. For that, experimental tests and finite element analysis (FEA) simulation were conducted to determine the collapse behaviour of these various symmetrical geometric tubes. First, the quasi-static lateral compression test was conducted on square and cylindrical tubes experimentally and by FEA simulation method by using INSTRON Universal Testing Machine and ABAQUS software respectively. Both results were compared to validate the FEA simulation results. Then, the validated FEA simulation method was performed for these various symmetrical geometric tubes to determine their deformation behaviour and energy-absorption performance and then to validate the newly mathematical model. The comparison between the experiment and FEA simulation had shown good agreement. The simulation study showed that square and symmetric hexagonal tubes deformed with 1 phase of plastic deformation, symmetric octagonal and decagonal tubes deformed with 2 phases of plastic deformation, symmetric dodecagonal and tetra-decagonal tubes deformed with 3 phases of plastic deformation. It was determined that, the general mathematical model had succeeded to predict the deformation behaviour of various symmetric geometrical shapes for both cases but discrepancy occurred for certain specimens due to sudden high peak at the last phase and small angle difference for neighbouring sides. The energy – absorption performance analyses for different types of symmetric geometrical tubes had shown that symmetric hexagonal tube produced the best energy-absorption with high total energy absorption, low yield stress and long stroke without any sudden jump force.

ABSTRAK

Tujuan kajian ini adalah untuk membangunkan model matematik ramalan umum bagi tingkah laku ubahan-bentuk untuk berbagai tiub bergeometrik simetri di bawah mampatan sisian antara dua plat tegar rata. Model matematik dicadangkan berdasarkan model tegar, plastik sempurna dan kaedah tenaga saksama. Model matematik ini terbahagi kepada dua kes iaitu 'Kes 1' dan 'Kes 2' berdasarkan kepada bentuk geometrik tiub. Kes ' 1' adalah untuk bentuk tiub dengan bilangan sisi 6, 10, 14 dan seterusnya seperti tiub heksagon, dekagon dan tetra-dekagon. Manakala, 'Kes 2' bagi bentuk tiub dengan bilangan sisi 4, 8, 12 dan seterusnya seperti tiub segi empat sama, oktagon dan dodekagon. Ramalan atau andaian yang digunakan dalam model matematik ini adalah bahawa tiub akan mengalami ubah-bentuk fasa demi fasa semasa ubahan-bentuk plastik. Untuk mencapai tujuan ini, tingkah-laku ubah-bentuk dan prestasi serapan-tenaga tiub-tiub berbagai bentuk geometrik perlu ditentukan. Bentuk-bentuk geometrik tiub yang dikaji termasuk tiub segi empat sama, heksagon, oktagon, dekagon, dodekagon dan tetra-dekagon. Untuk itu, simulasi analisis unsur terhingga (FEA) dan ujian eksperimen telah dijalankan untuk menentukan tingkah-laku keruntuhan tiub-tiub bergeometrik simetri tersebut. Pertama, ujian mampatan sisian separa statik dijalankan ke atas tiub segi empat sama dan tiub silinder secara eksperimen menggunakan mesin ujikaji universal INSTRON dan secara simulasi FEA menggunakan perisian ABAQUS. Kedua-dua keputusan dibandingkan untuk mengesahkan keputusan simulasi FEA. Kemudian, kaedah simulasi FEA yang telah disahkan dilakukan ke atas kesemua tiub-tiub bergeometrik simetri tersebut untuk menentukan tingkah laku ubahan-bentuk dan prestasi serapan-tenaga bentuk-bentuk tersebut dan kemudian untuk mengesahkan model baru matematik. Perbanding antara eksperimen dan simulasi FEA telah menunjukkan perjanjian yang baik. Kajian simulasi menunjukkan bahawa tiub segiempat sama dan tiub heksagon simetri mengalami ubah-bentuk dengan 1 fasa pada ubahan-bentuk plastik, tiub oktagon simetri dan tiub dekagon simetri mengalami ubah-bentuk dengan 2 fasa pada ubahan-bentuk plastik, tiub dodekagon simetri dan tiub tetradekagon simetri mengalami ubah-bentuk dengan 3 fasa pada ubahan-bentuk plastik. Telah dipastikan bahawa model matematik umum telah berjaya untuk meramalkan tingkah-laku ubahan-bentuk pelbagai bentuk tiub bergeometrik simetri bagi kedua-dua kes. Walau bagaimanapun, berlaku percanggahan pada spesimen tertentu disebabkan kemunculan puncak tinggi secara mendadak di fasa terakhir dan perbezaan sudut yang kecil pada sisi-sisi yang berjiran. Analisis prestasi serapan-tenaga pada tiub-tiub bergeometrik simetri yang berbeza telah menunjukkan bahawa tiub heksagon simetri menghasilkan serapan-tenaga terbaik dengan jumlah serapan-tenaga yang tinggi, kadar hasil yang rendah dan strok yang panjang tanpa mana-mana peningkatan mendadak pada kuasa.

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LIST OF ABBREVIATIONS

θ_i - The angle between the oblique side and the vertical axis at phase-i

δ - The displacement.

κ - The bending curvature

κ_e - The maximum elastic curvature

ν - Poisson’s Ratio

σ - Stress (N/m^2)

σ_{Y} - Yield stress (N/m^2)

σ_{UTS} - Ultimate stress (N/m^2)

$\in$ - No. of element

$\mathbb{Z}$ - Integers

ε - Normal strain (mm)

ε_{y} - Yield strain (mm)

ε_{f} - Fracture strain (mm)

E - Young's modulus (N/m^2)

E_p - Hardening modulus (N/m^2)

E_{in} - Input energy / external energy (kJ)

W^e - Elastic strain energy (kJ)

D - Plastic strain energy (kJ)

F - Force (N)

F^o - Lower bound of the actual limit loads (N)

F^* - Upper bound of the actual limit loads (N)

F_s - Actual limit loads (N)

H_i - Vertical height of the oblique side at phase-i, (mm)

K - Material constants

L - Length (m)

M - Bending moment (N)

M_e - Maximum elastic bending moment (N)

M_p - The fully plastic bending moment (N)

N - Normal forces (N)

Y - The yield stress of the material (N/m^2)

U1 - Displacement component in the 1-direction (mm)

U2 - Displacement component in the 2-direction (mm)

U3 - Displacement component in the 3-direction (mm)

UR1 - Rotational displacement component about the 1-direction

UR2 - Rotational displacement component about the 2-direction

UR3 - Rotational displacement component about the 3-direction

b - The width of the tube (mm)

h - The thickness of the tube (mm)

m - Number of phases

n - Number of sides

q - Hardening exponent

CHAPTER 1

INTRODUCTION

1.1 Background of the Research

In the modern era of lives, transportation is one of the main needs to travel from one location to another location and to deliver goods. Due to the advanced technology of the modern world, the vehicles could be produced in a massive volume. In Malaysia, the number of vehicles registered in the year of 2011 was 21,311,630 increased by more than 1 million from the year of 2010 (Royal Malaysia Police, 2012). Moreover, vehicles can also have very high speeds. There are also a lot of heavy vehicles like lorries and trucks on the road. The increasing number of vehicles with high speeds and massive weight will lead to a more severe damage to the people and environment if traffic accident occurs.

The number of people killed and injured due to the road accident is reported to be increasing year by year. World Health Organization (WHO) reported around 1.3 million people are killed in road traffic collisions worldwide every year (WHO, 2009). Furthermore, the number of injuries or disabilities is estimated between 20 and 50 million people worldwide every year. The European Union (EU) with the number of motor vehicles is nearly half of the about 500 million population reported the numbers of injuries and deaths from road accidents are 1200, and 34,500 respectively each year (European Commission, 2011). The United States of America (USA) with 309 million population and 256 million registered motorised vehicles in 2008, reported 33,808 deaths due to road accidents (National Highway Traffic Safety Administration, 2010). In Malaysia, nearly 7000 deaths and over 25,000 injuries have been reported in 2011due to road accidents (Royal Malaysia Police, 2012). Hence, road traffic fatalities, disabilities, and injuries have become a major global public health issue. Due to these associated increases, society has become more aware and concerned for the safety aspects of transportation.

This has led researchers in the last few decades to study and develop impact protection systems to prevent and reduce the effects of collisions. These safety systems can be divided into two types i.e. active and passive safety systems (Johnson and Mamalis, 1978). The function of active safety systems is to prevent collision to happen. Some of the examples of active safety system are the application of electronic control systems to improve drivers' visibility, improved vehicle handling devices and anti-lock-braking systems (ABS). On the other hand, the function of passive safety systems is to reduce the collision effects to the vehicles and occupants by limiting the level of deceleration and dissipating the kinetic energy during impact in the controlled manner. Some of the examples of passive safety systems are the