COMPUTATIONAL ALGORITHM FOR INDOOR MOBILE ROBOT PATH SEARCHING VIA TOR 9-POINT LAPLACIAN ITERATION FAMILY

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COMPUTATIONAL ALGORITHM FOR INDOOR MOBILE ROBOT PATH SEARCHING VIA TOR 9-POINT LAPLACIAN ITERATION FAMILY

LING WAI KIAT

Thesis submitted to the Centre for Graduate Studies, Universiti Pertahanan Nasional Malaysia, in fulfilment of the requirements for the Degree of Master of Science (Mathematics)

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ABSTRACT

Mobile robot path navigation is a subject that has been a crucial study in the robotics field. The ability of robots to navigate successfully demands high performance path searching algorithms. The room for improvements to raise the potential of autonomous path searching is far from seeing its limits. Any approach that bears potential needs to be explored. Thus, the objective of this study is to investigate the performance of a combined set of numerical techniques based on Laplacian potential values in producing path searching algorithm for mobile robots. This technique set comprised of Quarter-Sweep approach (QS) for complexity reduction, family of Two-parameter Overrelaxation iterative method (TOR) for reducing the cost of computations, and 9-Point Laplacian operator (9P) for improving computational efficiency. This study is conducted in a robot path searching simulator, namely Robot 2D Simulator. The configuration space in the simulator is set up accordingly to resemble a 2D heat transfer environment, and numerical analysis via Laplace's equation can be applied to the resulting heat distributions. The proposed numerical techniques are used to solve and obtain Laplacian potential values, and their corresponding path searching algorithms are tested for their performance. Results show that the proposed numerical techniques outperform their predecessors. Integration of family of TOR-9P iterative method and QS approach, i.e., Quarter-Sweep Two-parameter Overrelaxation 9-Point Laplacian method (QSTOR-9P) has produced the best results. The method succeeded in reducing the number of iteration and CPU time to approximately 62.27% to 87.64% and 83.30% to 95.20%,

respectively compared to the standard. Thus, it is concluded that the proposed numerical techniques have the potential to enable higher path searching performance.

ABSTRAK

Navigasi laluan robot boleh gerak adalah subjek yang penting dalam bidang penyelidikan robotik. Keupayaan robot boleh gerak memerlukan algoritma pencarian laluan yang berprestasi tinggi. Setakat ini, kajian bagi penambahbaikan terhadap keupayaan pencarian laluan masih perlu dilakukan. Malah, sebarang kaedah yang mempunyai potensi yang tinggi perlulah dikaji dengan lebih lanjut lagi. Sehubungan itu, objektif kajian ini adalah untuk mengkaji keupayaan gabungan set bagi kaedah berangka berdasarkan nilai potensi Laplacian terhadap penjanaan algoritma pencarian laluan robot boleh gerak. Kaedah berangka ini merangkumi pengunaan pendekatan Sapuan-Suku (QS) untuk mengurangkan kerumitan, kumpulan kaedah lelaran Pengenduran Berlebihan Dua-parameter (TOR) untuk mengurangkan kos pengiraan, dan 9-Titik Laplacian (9P) untuk meningkatkan kecekapan pengiraan. Kajian ini dilakukan dengan menggunakan sebuah simulasi pencarian laluan robot boleh gerak yang dinamakan Simulator Robot 2D. Ruang kerja dua dimensi dalam simulasi ini dibina bagi menyerupai sebuah persekitaran meliputi proses pemindahan haba, dan analisis berangka ke atas taburan haba yang terhasil boleh dilakukan melalui persamaan Laplace. Suatu kaedah berangka telah dicadangkan dalam kajian ini bertujuan untuk menyelesaikan dan mendapatkan nilai potensi Lapacian, dan prestasi algoritma pencarian laluan yang terhasil telah diuji. Kaedah tersebut berjaya memberikan keputusan yang cemerlang dalam menghasilkan navigasi laluan dan mengatasi prestasi kaedah terdahulu. Penyepaduan kumpulan kaedah lelaran TOR-9P dan pendekatan QS, iaitu QSTOR-9P telah menghasilkan keputusan yang baik. Kaedah ini telah berjaya menurunkan bilangan lelaran dan masa CPU masing-masing

daripada 62.27% kepada 87.64% dan 83.30% kepada 95.20%. Oleh itu, berdasarkan keputusan yang diperoleh daripada kajian ini, kaedah berangka yang dicadangkan dalam kajian ini mempunyai potensi bagi membolehkan prestasi pencarian laluan yang lebih tinggi.

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APPROVAL

The Examination Committee has met on **21 April 2021** to conduct the final examination of **LING WAI KIAT** on his degree thesis entitled **'Computational Algorithm for Indoor Mobile Robot Path Searching via TOR 9-Point Laplacian Iteration Family'.**

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TABLE OF CONTENTS

TITLE		PAGE
ABSTRACT		**ii**
ABSTRAK		**iv**
ACKNOWLEDGEMENTS		**vi**
APPROVAL		**vii**
APPROVAL		**viii**
DECLARATION OF THESIS		**ix**
TABLE OF CONTENTS		**x**
LIST OF TABLES		**xiii**
LIST OF FIGURES		**xiv**
LIST OF ABBREVIATIONS		**xvii**
LIST OF SYMBOLS		**xx**
LIST OF APPENDICES		**xxi**
CHAPTER 1	**INTRODUCTION**	**1**
	1.1 Background	1
	1.2 Path Searching Problem	5
	1.3 Path Searching Using Laplacian Operator	6
	1.4 The Heat Transfer Analogy and The Configuration Space	9
	1.5 Problem Statements	10
	1.6 Research Questions	11
	1.7 Objectives of Study	12
	1.8 Scope and Restrictions	12
	1.9 Outline	14
	1.10Concluding Remarks	15
CHAPTER 2	**LITERATURE REVIEW**	**17**
	2.1 Introduction on Autonomous Path Searching	17
	2.2 Path Searching for Mobile Robot	18
	2.2.1 Local Path Searching	30
	2.2.2 Global Path Searching	31
	2.2.3 Gradient Descent Search Technique	32
	2.3 Applications of Laplacian Operator in Robotics	33
	2.4 Computation of Laplacian Operator	35
	2.4.1 The Iterative Method in Linear System Computation	36
	2.4.2 The Overrelaxation Iterative Methods	38
	2.5 Path Searching Strategy	40
	2.5.1 Heat Transfer Analogy	41
	2.5.2 Harmonic Function	42

2.5.3 The Configuration Space 43
2.5.4 Path Construction 45
2.6 Half-Sweep (HS) and Quarter-Sweep (QS) Concepts 45
2.7 Research Motivations 48
2.8 Concluding Remarks 49

CHAPTER 3 THE ITERATIVE METHODS FOR ROBOT PATH SEARCHING 51
3.1 Introduction 51
3.2 Iterative Method as Solutions to Laplace's Equation 52
3.3 Family of 5-Point Laplacian Operator 54
3.3.1 Formulations of 5-Point Laplacian Operator 54
3.3.2 5-Point Finite Difference Approximation 60
3.3.3 Formulation of 5-Point Laplacian Full-Sweep (FS) Iterative Methods 68
3.3.4 Formulation of 5-Point Laplacian Half-Sweep (HS) Iterative Methods 72
3.3.5 Formulation of 5-Point Laplacian Quarter-Sweep (QS) Iterative Methods 76
3.4 Family of 9-Point Laplacian Operator 81
3.4.1 Formulations of 9-Point Laplacian Operator 81
3.4.2 9-Point Finite Difference Approximation 87
3.4.3 Formulation of 9-Point Laplacian Full-Sweep (FS) Iterative Methods 88
3.4.4 Formulation of 9-Point Laplacian Half-Sweep (HS) Iterative Methods 92
3.4.5 Formulation of 9-Point Laplacian Quarter-Sweep (QS) Iterative Methods 96
3.5 Concluding Remarks 102

CHAPTER 4 SIMULATION RESULTS OF ITERATIVE METHODS 104
4.1 Introduction 104
4.2 Experimental Setup 105
4.3 Simulation of 5-Point Iterative Methods 108
4.3.1 Results and Discussions 109
4.4 Simulation of Family of 9-Point Iterative Methods 117
4.4.1 Results and Discussions 117
4.5 Analysis of Computational Complexity 125
4.6 Concluding Remarks 129

CHAPTER 5 CONCLUSION AND RECOMMENDATIONS 135
5.1 Summary 135
5.2 Future Works and Recommendations 136
5.3 Conclusions 137

REFERENCES 140

APPENDICES **153**
BIODATA OF STUDENT **159**
LIST OF PUBLICATIONS **160**

LIST OF TABLES

TABLE NO.	TITLE	PAGE
Table 2.1	Path searching algorithms using GDS for local, global or local/global hybrid of path searching	29
Table 4.4	The reduction percentages range based on number of iterations and CPU time achieved via 5-Point iterative method path searching algorithms	115
Table 4.5	The reduction percentages range achieved via HS and QS iterative methods compared to corresponding FS iterative methods (i.e., FSSOR-5P, FSAOR-5P, and FSTOR-5P) based on number of iterations and CPU time	116
Table 4.9	The reduction percentages range based on number of iterations and CPU time achieved via 9-Point iterative method path searching algorithms	123
Table 4.10	The reduction percentages range achieved via HS and QS methods compared to corresponding FS method (i.e., FSSOR-9P, FSAOR-9P, and FSTOR-9P) based on number of iterations and CPU time	124
Table 4.11	Arithmetic operation numbers per iteration for FSSOR-5P, HSSOR-5P, QSSOR-5P, FSSOR-9P, HSSOR-9P, and QSSOR-9P iterative methods	126
Table 4.12	Arithmetic operation numbers per iteration for FSAOR-5P, HSAOR-5P, QSAOR-5P, FSAOR-9P, HSAOR-9P, and QSAOR-9P iterative methods	127
Table 4.13	Arithmetic operation numbers per iteration for FSTOR-5P, HSTOR-5P, QSTOR-5P, FSTOR-9P, HSTOR-9P, and QSTOR-9P iterative methods	128
Table 4.14	Arithmetic operation numbers for direct calculation method of remaining nodal points	129

LIST OF FIGURES

FIGURE NO.	TITLE	PAGE
Figure 1.1	Unimate	2
Figure 1.2	A disinfection robot developed to fight against COVID-19 in Qingdao, East China's Shandong province	3
Figure 1.3	Battlefield Extraction-Assist Robot (BEAR)	4
Figure 1.4	Roomba, an autonomous vacuum	5
Figure 1.5	Pepper, an entertainment robot	5
Figure 1.6	Robonaut 2, a helper robot in the international space station	5
Figure 1.7	Curiosity, a robotic rover deployed to Mars	5
Figure 2.1	Mobile robot global path searching workflow	31
Figure 2.2	Illustration of a heat sink in the configuration space	42
Figure 2.3	The configuration spaces for mobile robot path searching simulation	44
Figure 2.4	Nodal points considered (dots coloured in black) for iterative computation in configuration space for (a) FS, (b) HS, and (c) QS respectively	47
Figure 3.1	Standard 5-Point FS method nodal points computational structure	56
Figure 3.2	Standard 5-Point HS method in a portion of configuration space	56
Figure 3.3	Rotated 5-Point HS method of nodal points computational structure	57
Figure 3.4	Rotated 5-Point HS method in a portion of configuration space	58
Figure 3.5	Standard 5-Point QS method of nodal points computational structure	59

Figure 3.6 Standard 5-Point QS method placement in a portion of configuration space 59

Figure 3.7 Nodal points computational scheme in a portion of the configuration space grid for (a) FS, (b) HS, and (c) QS respectively 68

Figure 3.8 Extended 9-Point FS method nodal points computational structure 81

Figure 3.9 Extended 9-Point FS method placement in a portion of configuration space 82

Figure 3.10 Rotated 9-Point HS method nodal points computational structure 84

Figure 3.11 Extended 9-Point HS method placement in a portion of configuration space 84

Figure 3.12 Extended 9-Point QS method nodal points computational structure 85

Figure 3.13 Extended 9-Point QS method placement in a portion of configuration space 86

Figure 3.14 Overview of the existing and proposed iterative methods for solving mobile robot path searching problem 103

Figure 4.1 Robot 2D Simulator interface sample for testing FSSOR-5P path searching performance in Environment 1 of size 600×600 108

Figure 4.2 Performance of path searching algorithms FSTOR-5P, HSTOR-5P, and QSTOR-5P in Environment 1 based on number of iterations 111

Figure 4.3 Performance of path searching algorithms FSTOR-5P, HSTOR-5P, and QSTOR-5P in Environment 1 based on CPU time (in seconds) 114

Figure 4.4 Performance of path searching algorithms FSTOR-9P, HSTOR-9P, and QSTOR-9P in Environment 1 based on number of iterations 119

Figure 4.5 Path searching performance of algorithms FSTOR-9P, HSTOR-9P, and QSTOR-9P in Environment 1 based on CPU time (in seconds) 122

Figure 4.6 The generated paths in Robot 2D Simulator for Environment 1 131

Figure 4.7 The generated paths in Robot 2D Simulator for Environment 2 132

Figure 4.8 The generated paths in Robot 2D Simulator for Environment 3 133

Figure 4.9 The generated paths in Robot 2D Simulator for Environment 4 134

LIST OF ABBREVIATIONS

ACO	-	Ant Colony Optimisation
ANN	-	Artificial Neural Network
AOR	-	Accelerated Overrelaxation iterative method
APF	-	Artificial Potential Fields
ASV	-	Autonomous Surface Vehicle
BEAR	-	Battlefield Extraction-Assist Robot
CD	-	Cell Decomposition
CPU	-	Central Processing Unit
DARPA	-	Defense Advanced Research Projects Agency
EKF	-	Extended Kalman Filter
FDM	-	Finite Difference Method
FS	-	Full Sweep
FSAOR-5P	-	Full Sweep Accelerated Overrelaxation 5-Point Laplacian operator
FSAOR-9P	-	Full Sweep Accelerated Overrelaxation 9-Point Laplacian operator
FSSOR-5P	-	Full Sweep Successive Overrelaxation 5-Point Laplacian operator
FSSOR-9P	-	Full Sweep Successive Overrelaxation 9-Point Laplacian operator
FSTOR-5P	-	Full Sweep Two-parameter Overrelaxation 5-Point Laplacian operator
FSTOR-9P	-	Full Sweep Two-parameter Overrelaxation 9-Point Laplacian operator
GA	-	Genetic Algorithm
GDS	-	Gradient Descent Search
HPF	-	Harmonic Potential Field
HS	-	Half Sweep

HSAOR-5P	-	Half Sweep Accelerated Overrelaxation 5-Point Laplacian operator
HSAOR-9P	-	Half Sweep Accelerated Overrelaxation 9-Point Laplacian operator
HSSOR-5P	-	Half Sweep Successive Overrelaxation 5-Point Laplacian operator
HSSOR-9P	-	Half Sweep Successive Overrelaxation 9-Point Laplacian operator
HSTOR-5P	-	Half Sweep Two-parameter Overrelaxation 5-Point Laplacian operator
HSTOR-9P	-	Half Sweep Two-parameter Overrelaxation 9-Point Laplacian operator
LU	-	Lower Upper
NASA	-	National Space Agency
NC-RRT	-	Node Control Rapidly-exploring Random Tree
OCPB	-	Operative Critical Point Bug
PDE	-	Partial Differential Equation
PRM	-	Probabilistic Roadmap
PSO	-	Particle Swarm Optimisation
QS	-	Quarter Sweep
QSAOR-5P	-	Quarter Sweep Accelerated Overrelaxation 5-Point Laplacian operator
QSAOR-9P	-	Quarter Sweep Accelerated Overrelaxation 9-Point Laplacian operator
QSSOR-5P	-	Quarter Sweep Successive Overrelaxation 5-Point Laplacian operator
QSSOR-9P	-	Quarter Sweep Successive Overrelaxation 9-Point Laplacian operator
QSTOR-5P	-	Quarter Sweep Two-parameter Overrelaxation 5-Point Laplacian operator

QSTOR-9P	-	Quarter Sweep Two-parameter Overrelaxation 9-Point Laplacian operator
RAM	-	Random Access Memory
RRT	-	Rapidly-exploring Random Tree
SA	-	Simulated Annealing
SLAM	-	Simultaneous Localisation and Mapping
SOR	-	Successive Overrelaxation iterative method
TCM	-	Time Critical Mobility
TOR	-	Two-parameter Overrelaxation iterative method
TS	-	Tabu Search
UAV	-	Unmanned Aerial Vehicle

LIST OF SYMBOLS

ω	-	SOR parameter
ω'	-	AOR parameter
ω''	-	TOR parameter
U, u	-	Potential value
N	-	Size of environment
ε	-	Convergence criterion

LIST OF APPENDICES

APPENDICES	TITLE	PAGE
A-1	Table 4.1: Performance of the 5-Point iterative method path searching algorithms based on number of iterations with corresponding reduction percentages	153
A-2	Table 4.2: Maximum absolute error from the iteration process of 5-Point iterative method path searching algorithms	154
A-3	Table 4.3: Performance of the 5-Point iterative method path searching algorithms based on CPU time (in seconds) with corresponding reduction percentages	155
A-4	Table 4.6: Performance of the 9-Point iterative method path searching algorithms based on number of iterations with corresponding reduction percentages	156
A-5	Table 4.7: Maximum absolute error from the iteration process of 9-Point iterative method path searching algorithms	157
A-6	Table 4.8: Performance of the 9-Point iterative method path searching algorithms based on CPU time (in seconds) with corresponding reduction percentages	158

CHAPTER 1

INTRODUCTION

1.1 Background

A robot is essentially a machine that functions by computing pre-installed programming to make decisions. It usually consists of three main components, which are the sensory element, control system, and actuators. The sensory element of a robot usually consists of various sensors for detecting image, sound, and obstacles. The sensor will transmit input information into the second component that is the control system. Here, the input will be used for the decision-making process of the robot. Finally, the third component of the robot, which is the actuator will produce the actuation action as an output from the robot decision-making process.

Robots have been introduced in earlier years to carry out repetitive tasks that can be automated. The first robot employed in the manufacturing industry was Unimate (Spectrum, 2018), which was basically an arm of hydraulic manipulator that was able to perform repetitive tasks, shown in Figure 1.1. It was first installed in the General Motors plant in New Jersey in 1961. The inventor of Unimate named George Devol described the Unimate as 'an autonomous machine that could store step-by-step digital commands to move parts in factory'. Unimate received warm welcomes from the manufacturing industry and was mostly used by car manufacturers for automation of metalworking and welding processes. From here, the operational basics of a robot can be observed, which is to perform under the governing digital instructions pre-installed in the robot.

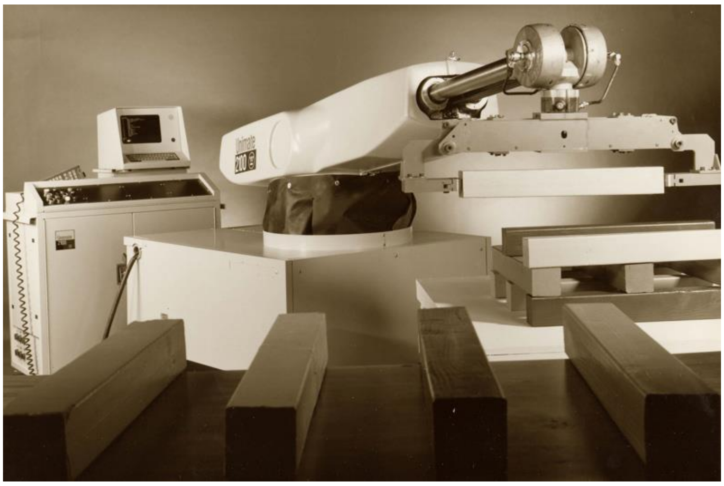

Figure 1.1 Unimate

Following the success of Unimate, robots become increasingly significant in human life. The application of robots eventually receives popular demand and is integrated into various industries. These industries include agriculture, education, civil defence, military, and medical. In recent events, during the global health emergency in 2020 due to the COVID-19 outbreak, robots were deployed in the medical facilities in China (Figure 1.2). They were mostly used as automatic disinfectors, as part of the effort to fight the virus outbreak (Network, 2020).

Figure 1.2 A disinfection robot developed to fight against COVID-19 in Qingdao, East China's Shandong province

Perhaps, one of the most active promoters of the robotics field nowadays is the military. As robots were used in industries to reduce hard human labours, the military was utilising robots mostly to reduce human casualties on the battlefield. A division of US Department of Defence, known as the Defence Advanced Research Projects Agency (DARPA) is one of the biggest pioneers in military robotics research agency and has been in the field since 1958. The agency has been actively studying robotics in the military field, some of the studies and innovations include Unmanned Aerial Vehicle (UAV) surveillance drones, exoskeletons, and most recently the Battlefield Extraction-Assist Robot (BEAR). BEAR is a humanoid robot with the main objective of rescuing wounded soldiers from the battlefield while reducing human deployment, thus reducing casualties (Figure 1.3).